III.

NOTE ON GRASSMANN'S CALCULUS OF EXTENSION.

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The last "Mathematische Annalen" contains a paper by H. Grassmann, on the application of his calculus of extension to Mechanics.

He adopts the quaternion addition of vectors. But he has two multiplications, internal and external, just as the principles of logic require.

The *internal* product of two vectors, v_1 and v_2, is simply what is written in quaternions as $- S.\, v_1\, v_2$. He writes it $[v_1 \mid v_2]$. So that

$$[v_1 \mid v_2] = [v_2 \mid v_1],$$

$$v^2 = (Tv)^2.$$

The *external* product of two vectors is the parallelogram they form, account being taken of its plane and the direction of running round it, which is equivalent to its *aspect*. We therefore have : —

$$[v_1 v_2] = v_1 v_2 \sin \angle{}^{v_1}_{v_2}.\ \mathrm{I}.$$

$$[v_1 v_2] = - [v_2 v_1], \qquad v^2 = o,$$

where I is a new unit. This reminds me strongly of what is written in quaternions as $- V(v_1 v_2)$. But it is not the same thing in fact, because $[v_1 v_2] v_3$ is a solid, and therefore a new kind of quantity. In truth, Grassman has got hold (though he did not say so) of an eight-fold algebra, which may be written in my system as follows : —

Three Rectangular Vectors.

$$i = M : A - B : Z + C : Y + X : N$$

$$j = M : B - C : X + A : Z + Y : N$$

$$k = M : C - A : Y + B : X + Z : N$$

Three Rectangular Planes.

$$I = M : X + A : N$$

$$J = M : Y + B : N$$

$$K = M : Z + C : N$$

One Solid.

$$V = M : N$$

Unity.

$$1 = M : M + A : A + B : B + C : C$$
$$+ N : N + X : X + Y : Y + Z : Z$$

This unity might be omitted.

The relation of the two multiplications is exceedingly interesting. The system seems to me more suitable to three dimensional space, and also more natural than that of quaternions. The simplification of mechanical formulæ is striking, but not more than quaternions would effect, that I see.

By means of eight rotations through two-thirds of a circumference, around four symmetrically placed axes, together with unity, all distortions of a particle would be represented linearly. I have therefore thought of the nine-fold algebra thus resulting.