



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

III.

NOTE ON GRASSMANN'S CALCULUS OF EXTENSION.

BY C. S. PEIRCE.

Read Oct. 10, 1877.

THE last "Mathematische Annalen" contains a paper by H. Grassmann, on the application of his calculus of extension to Mechanics.

He adopts the quaternion addition of vectors. But he has two multiplications, internal and external, just as the principles of logic require.

The *internal* product of two vectors, v_1 and v_2 , is simply what is written in quaternions as $-S.v_1 v_2$. He writes it $[v_1 | v_2]$. So that

$$[v_1 | v_2] = [v_2 | v_1],$$

$$v^2 = (Tv)^2.$$

The *external* product of two vectors is the parallelogram they form, account being taken of its plane and the direction of running round it, which is equivalent to its *aspect*. We therefore have:—

$$[v_1 v_2] = v_1 v_2 \sin \angle_{v_2}^{v_1}. I.$$

$$[v_1 v_2] = -[v_2 v_1], \quad v^2 = 0,$$

where I is a new unit. This reminds me strongly of what is written in quaternions as $-V(v_1 v_2)$. But it is not the same thing in fact, because $[v_1 v_2]v_3$ is a solid, and therefore a new kind of quantity. In truth, Grassman has got hold (though he did not say so) of an eight-fold algebra, which may be written in my system as follows:—

Three Rectangular Vectors.

$$i = M : A - B : Z + C : Y + X : N$$

$$j = M : B - C : X + A : Z + Y : N$$

$$k = M : C - A : Y + B : X + Z : N$$

Three Rectangular Planes.

$$I = M : X + A : N$$

$$J = M : Y + B : N$$

$$K = M : Z + C : N$$

One Solid.

$$V = M : N$$

Unity.

$$\begin{aligned} 1 = & M : M + A : A + B : B + C : C \\ & + N : N + X : X + Y : Y + Z : Z \end{aligned}$$

This unity might be omitted.

The relation of the two multiplications is exceedingly interesting. The system seems to me more suitable to three dimensional space, and also more natural than that of quaternions. The simplification of mechanical formulæ is striking, but not more than quaternions would effect, that I see.

By means of eight rotations through two-thirds of a circumference, around four symmetrically placed axes, together with unity, all distortions of a particle would be represented linearly. I have therefore thought of the nine-fold algebra thus resulting.