3. The opposite leg of a right triangle is the leg which does not touch the vertex of the angle that is named in the trig function.

4. The adjacent leg of a right triangle is the leg which does touch the vertex of the angle that is named in the trig function; for example:

When evaluating the trig functions for angle A in this right triangle, leg y is the opposite leg for angle A because it does not touch point A; however, leg x is the adjacent leg for angle A because it does touch point A; the hypotenuse is side z.

In the same right triangle, leg x is the opposite leg for angle B because it does not touch point B; however, leg y is the adjacent leg for angle B because it does touch point B.

**NOTE:** The legs that is the opposite leg for angle A is the same leg that is the adjacent leg for angle B; and the leg that is the adjacent leg for angle A is the same leg that is the opposite leg for angle B; the hypotenuse is never considered as the opposite side nor as the adjacent side because it is not a leg.

5. Since trig functions are ratios, and ratios can be written as decimal numbers, trig functions are either converted to decimal numbers or left as radical expressions in lowest terms (for example, \(\frac{\sqrt{3}}{2}\) or .866); for example, in the right triangles above, if:

\[ z = 9, \ y = 7, \text{ and } x = \sqrt{121} = 11 \]  

\[ \sin A = \cos B = \frac{7}{9} = .7778 \]  
 \[ \cos A = \sin B = \frac{\sqrt{100}}{9} = .6285 \]  
 \[ \tan A = \frac{\sqrt{3}}{3} = 1.2374 \]  
 \[ \tan B = \frac{\sqrt{3}}{3} = .8081 \]

6. Using the trig function decimal number values to find or use angle measures requires either a trig function chart or a calculator with trig function options; for example, if you have found that sin \(\alpha = .7778\) then, by using either a trig chart or calculator, the measure of angle \(\alpha\) is about 51°.

<table>
<thead>
<tr>
<th>(\theta) degrees</th>
<th>(\text{sine})</th>
<th>(\text{cosine})</th>
<th>(\text{tangent})</th>
</tr>
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<tr>
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</tr>
</tbody>
</table>

If using a calculator, follow the calculator directions

C. Triangle Trig Applications

There are two basic ways in which trig functions are used with triangles: to find angle measures and to find side lengths.

1. Right Triangles

a. Finding Acute Angle Measures

To find the two acute angle measures when given two sides of a right triangle, it is easiest to find the length of the third side first; for example, in the following right triangle, if you know the length of any two sides, then you may use the Pythagorean Theorem (\(a^2 + b^2 = c^2\)) to find the length of the third side.

Once the three side lengths are found (it is not necessary to find the third side lengths in order to find the angle measures, but it is easier), then use the trig functions to find the degree measure of one acute angle; using the same right triangle above, the measure of \(\alpha\) can be found using any of the trig functions, so just pick one of them; for example:

The measure of the second acute angle may be found by simply subtracting the measure of the acute angle just found from 90° because the sum of the three angles of any triangle is 180°.

\[ \sin \alpha = \frac{3}{5} = .6000 \therefore \alpha = 36.87' \]  
\[ \beta = 90° - 36.87' = 53.13' \]
b. Finding Side Lengths

To find the side lengths of a right triangle when given only one side length and one acute angle, first, subtract the given acute angle measure from 90° because the sum of the three angles of any triangle is 180°. Second, use the trig functions to find the length of another side of the triangle; for example:

\[
\sin 35° = \frac{a}{14} \Rightarrow a = 14 \times 0.5736 = 8.0304
\]

Once two sides of the right triangle are known, the Pythagorean Theorem can be used to find the length of the third side.

c. Applying Sample Situations

i. Definition: The angle of elevation is the angle formed by a horizontal line (either real or imagined) and the line of sight looking up from the horizontal; for example:

**Problem:** Anna stood 4,800 feet from a rocket launching pad; she measured the angle of elevation as 73° when the rocket was at its highest point; if Anna measured the angle of elevation from a height of 5.5 feet, find the greatest height that the rocket reached.

\[
\tan 73° = \frac{h}{4800} \Rightarrow h = 4800 \times 3.2709 = 15700.32\text{ ft} \\
\text{Anna's height to her eyes is 5.5 feet.}
\]

ii. Definition: The angle of depression is the angle formed by a horizontal line (either real or imagined) and the line of sight looking down from the horizontal; for example:

**Problem:** A Coast Guard crew was flying a rescue mission in a helicopter; a member of the crew spotted a boat in trouble; this crewmember was looking down at about a 25° angle of depression; if the helicopter was about 300 feet above water level, how far did the helicopter have to travel to be above the boat?

\[
\tan 25° = \frac{d}{300} \Rightarrow d = 300 \times 0.4663 = 140\text{ ft}
\]

2. Oblique Triangles

Oblique triangles do not contain a right angle; therefore, any triangle that is not a right triangle is an oblique triangle.

a. Acute Triangles

Any acute triangle (triangle with all acute angles) can be separated into two right triangles by constructing a line segment from one of the vertices and perpendicular to the side opposite the vertex; for example, ΔABC can be formed into right triangles ABD and BCD by drawing line segment perpendicular to side AC.

Then the trig function definitions for right triangles can be applied as discussed above.

**NOTE:** Another option for solving acute triangles is to leave the triangles as they are (acute) and to apply the law of cosines or the law of sines, both of which are discussed at the top of the next column.

b. Obtuse Triangles

Any obtuse triangle (triangle with exactly one obtuse angle) can be converted into a right triangle by constructing a line segment from one of the vertices and perpendicular to the line containing the side opposite the vertex; for example, in ΔABC, ∠C is obtuse; extend side AC, then draw line segment perpendicular to the extension; the result is right ΔABD.

Then the trig function definitions for right triangles can be applied as discussed above.

**NOTE:** Another option for solving obtuse triangles is to leave the triangles as they are (obtuse) and to apply either the law of cosines or the law of sines, both of which are discussed at the top of the next column.

c. Law of Cosines

i. The law of cosines states that in a triangle ABC:

\[
a^2 = b^2 + c^2 - 2bc \cos \alpha \\
b^2 = a^2 + c^2 - 2ac \cos \beta \\
c^2 = a^2 + b^2 - 2ab \cos \gamma
\]

ii. When to apply the law of cosines

The law of cosines may be used either when all three sides of the triangle are known (SSS), or when only two side lengths and the measure of the angle formed by these two sides are known (SAS, that is, two sides and the included angle)

iii. Law of Sines

i. The law of sines states that in ΔABC (as indicated in ΔABC above in the law of cosines):

\[
\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}
\]

ii. When to apply the law of sines:

The law of sines may be used either when one side length and two angle measures are known (SSA), that is, one of the angles must be opposite the side or when two side lengths and one angle measure are known (SSA, that is, the angle must be opposite one of the two sides)

iii. Caution

When using the law of sines, occasionally there will be no solution; this is because not all combinations of angle measures and side lengths actually form triangles; remember that the third side of any triangle must have a length longer than the difference of the other two sides and shorter than the sum of these other two sides.

**TRIG WITH A UNIT CIRCLE**

A. Circles

1. Definitions

a. A circle is the set of points in a plane that are equidistant (the same distance) from one point, the center of the circle (which is not actually a point on the circle, but only the center)

b. A radius (r) is a line segment whose endpoints are a point on the circle and the center of the circle

c. A chord is a line segment whose endpoints are both points on the circle; all other points on the chord are points in the interior of the circle

d. A diameter (d) is a chord that contains the center of the circle

2. Central Angles

A central angle is an angle whose vertex is the center of a circle and whose sides contain points on the circle.

2. A central angle has the same degree measure as the arc of a circle

3. Degrees

a. One degree is 1/360 of the 360° contained in a complete circle; a degree may be subdivided into 60 minutes (written ‘60’); a minute may be subdivided into 60 seconds (written ‘60’)

b. The degree measure of an angle is the degree measure of the intercepted circular arc of the circle for which it is a central angle

4. Radians

a. One radian is the measure of a central angle that intercepts an arc equal in length to the radius of the circle

b. The radian measure of a central angle is the ratio of the circular arc length to the radius of the circle. Remember the distance around a circle is πd; for example:

5. Degree and Radian Conversions

a. A semicircle has a degree measure of 180° and a length equal to half the circle, πd/2r; the radian measure is the ratio between the circular arc length and the radius; therefore, the radian measure of a semicircle is πr/d; so:

b. Degree and radian conversions can be accomplished using the following conversions:

i. radian measure of the angle = degree measure of the angle \( \frac{\pi}{180}\) radians

ii. the radian measure of an angle = \( \pi \) (degree measure of the angle) \( \frac{180}{\pi}\) radians

iii. the degree measure of an angle = \( \frac{180}{\pi} \) (radian measure of the angle)
These functions are reciprocals:

\[ \frac{1}{\sin \alpha} = \csc \alpha \]
\[ \frac{1}{\cos \alpha} = \sec \alpha \]
\[ \frac{1}{\tan \alpha} = \cot \alpha \]

2. The equation of the unit circle is \( x^2 + y^2 = 1 \)

3. A point, \( P \), is on the unit circle if and only if the distance from the center of the circle to the point is equal to the radius of exactly one unit

4. The unit circle is symmetric with respect to the x-axis, the y-axis, and the origin; therefore, if point \( P = (a, b) \) is on the unit circle, then these points are also on the unit circle \((-a, b), (-a, -b), \) and \((a, -b)\); for example:

![Unit Circle Diagram](image)

5. The distance between any two points on the rectangular coordinate plane may be found by using the formula:

\[ \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]

6. The length of an arc of the unit circle is based on the circumference, \( \pi d = \pi 2 \) = \( \pi \) because, \( d = 2 \)

7. Points on the Unit Circle
   a. Points can be labeled using the appropriate order pair, \((x, y)\)
   b. Points can also be labeled using the circular arc length determined by the generated angle whose terminal side contains the point, \((x, y)\); for example:

   ![Unit Circle Points](image)

   c. Constructing a right triangle by drawing a perpendicular to the x-axis, and determining the side lengths of the triangle results in the following unit circle trig function definitions

   ![Unit Circle Trig Functions](image)

8. Unit circle trig function definitions (see the diagram above):

   \[ \text{Where } t = \text{ radians} \]
   \[ \alpha = \text{ degrees} \]

   \[ \sin \theta = \sin \alpha = y \]
   \[ \cos \theta = \cos \alpha = x \]
   \[ \tan \theta = \tan \alpha = \frac{y}{x} \quad \text{if } x \neq 0 \]

   \[ \csc \theta = \csc \alpha = \frac{1}{y} \quad \text{if } y \neq 0 \]
   \[ \sec \theta = \sec \alpha = \frac{1}{x} \quad \text{if } x \neq 0 \]
   \[ \cot \theta = \cot \alpha = \frac{x}{y} \quad \text{if } y \neq 0 \]

9. Frequently used angles and trig functions are indicated in the following chart

   ![Angle Chart](image)

10. Trig function graphs
   a. Graphing the values of the trig functions (indicated in the chart above) on the \( t \) (radians) -axis and the y-axis yields the following results

   ![Tangent Graph](image)

   i. The domain of the sine function is the set of real numbers; the range is the set of real numbers between -1 and 1, inclusively; i.e., \(-1 \leq y \leq 1\)

   ![Sine Graph](image)

   ii. Both the domain and the range of the cosine function are the same as the domain and the range of the sine function

   ![Cosine Graph](image)

   iii. The domain of the tangent function is the set of all real numbers except those values where the function is undefined and goes off asymptotically, such as \( \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \ldots \)

   The range is the set of all real numbers; the dashed lines are the vertical asymptotes

   ![Tangent Asymptotes](image)

   ![Tangent Graph with Asymptotes](image)

11. Periods of the functions
   a. A function, \( f \), is periodic if there is a positive number \( C \), such that \( f(t + C) = f(t) \) for all \( t \) in the domain of the function; this may also be stated using \( x \) in place of the \( t \) value. The smallest value of \( C \) called the period of the function; that is, the smallest value at which a function begins to repeat its range values, and thus repeat its graphing pattern, is the period of the function

   b. The period of the sine function, \( f(t) = \sin t \), is \( 2\pi \) because \( \sin(t + 2\pi) = \sin t \)

   c. The period of the cosine function, \( f(t) = \cos t \), is also \( 2\pi \) because \( \cos(t + 2\pi) = \cos t \)

   d. The period of the tangent function, \( f(t) = \tan t \), is \( \pi \) because \( \tan(t + \pi) = \tan t \)

   (NOTE: These periods can be observed in the graphs of the functions as indicated above)

   c. The period of a function, \( f(t) = \sin Bt \) is \( \frac{2\pi}{B} \); the effect of the value of \( B \) is that it stretches the graph out horizontally when \( 0 < B < 1 \) and shrinks the graph horizontally when \( B > 1 \)

   ![Sine Function Graph](image)

   ![Sine Function Graph with Period](image)

   (NOTE: The red section of the graph indicates one period of \( y = \sin x \); and the blue section is one period of \( y = \sin 2x \))

12. Amplitude
   a. The amplitude of a trig function, \( y = A \cdot \sin t \) or \( y = A \cdot \cos t \), can be defined as \( A \)

   Notice that when \( |A| > 1 \), the maximum and the minimum values of \( y \) equal \( A \), so the graph gets taller; likewise, when \( |A| < 1 \), the maximum and the minimum values of \( y \) equal \( A \), so the graph gets shorter; in the function \( f(t) = A \cdot \sin t \), the value of \( A \) does

   a. A generated angle (another type of angle often used in trigonometry) is a central angle with the vertex placed at the origin of the coordinate plane, and one of the two sides placed and kept on the positive x-axis, while the second side is rotated in either a clockwise or counterclockwise direction

   a. The side that does not rotate is called the generated angle

   i. The side that does rotate is called the terminal side

   c. Negative angles are formed when the terminal side rotates clockwise

   d. Positive angles are formed when the terminal side rotates counterclockwise
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NOTE TO STUDENT

Analytic Trig

A. Trig expressions contain trig functions and relationships, but no =, < or >; they may only be simplified; for example: (cos(x) + sin(x)) / (cos(x))

B. Trig equations contain trig functions and an equals sign; they may be solved to find the values that make them true; algebraic techniques, such as factoring, may be used to solve trig equations; for example:

C. Trig identities are true for all real numbers in the domain; they may be proven or verified; methods of proving or verifying identities include working the left side of the equation only until it is identical to the right side; working the right side until it is identical to the left side; or, working both sides until they are identical

D. Fundamental Trig Identities and Formulas

Reciprocals

Cofunctions

Basic Identities

Addition / Subtraction Formulas

Negatives

Double - Angle Formulas

Product - Sum Formulas

Credits

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